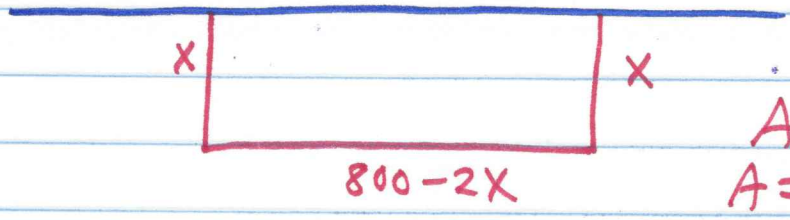
5.4 #9, 11, 13

9.



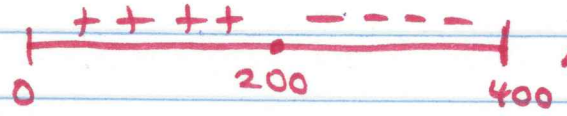
Maximize Area

$A = x(800-2x)$

$A = 800x - 2x^2$

$A'(x) = 800 - 4x \quad A' = 0 \quad 0 = 800 - 4x$

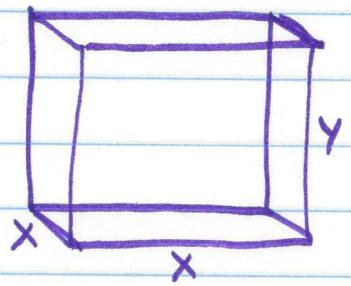
$x = 200 \quad A'(200) = 0$



$A(200) = 200(800 - 2(200)) = 200(400) = 80000$

$A(x)$ has max area of $A = 80000\ m^2$ at $x = 200\ m$ since $A'(x)$ changes from Pos to Neg.

11.



Minimize Weight

Minimize Surface Area

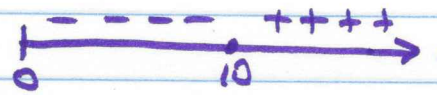
$V = 500\ ft^3$

$x^2y = 500 \quad y = \frac{500}{x^2}$

Area $A = x^2 + 4xy \quad A = x^2 + 4x\left(\frac{500}{x^2}\right)$

$A = x^2 + \frac{2000}{x} \quad A' = 2x - \frac{2000}{x^2} = \frac{2x^3 - 2000}{x^2}$

$A' = 0 \quad 0 = \frac{2x^3 - 2000}{x^2} \quad 0 = 2x^3 - 2000 \quad x = 10$



$A(10) = 100 + \frac{2000}{10} = 300$

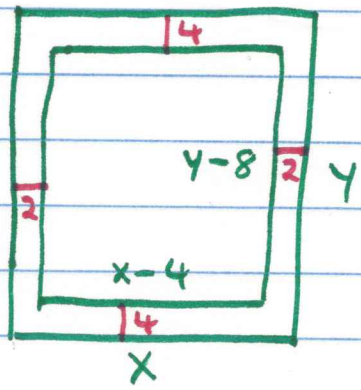
11. Continued

If $x=10$, then $y = \frac{500}{10^2} = 5$

If $x = 10$ ft, $y = 5$ ft, and $A = 300$ ft^2

The surface Area is a max of $A = 300$ ft^2 at $x = 10$ ft and $y = 5$ ft since $A'(x)$ changes from Neg. to Pos. at $x = 10$ ft.

13.



Minimize Amount of Paper

$50 = (x-4)(y-8)$

$\frac{50}{x-4} = y-8$

$y = \frac{50}{x-4} + 8$

Area of entire paper is $A = xy$

Minimize A

$A = xy = x\left(\frac{50}{x-4} + 8\right) = \frac{50x}{x-4} + 8x$

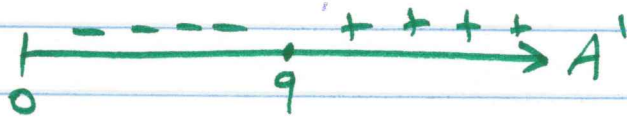
$A' = \frac{50(x-4) - 50x(1)}{(x-4)^2} + 8 = \frac{50x - 200 - 50x + 8(x-4)^2}{(x-4)^2}$

$= \frac{-200 + 8x^2 - 64x + 128}{(x-4)^2} = \frac{8x^2 - 64x - 72}{(x-4)^2}$

$A' = 0 \quad 0 = \frac{8x^2 - 64x - 72}{(x-4)^2} \quad 0 = 8x^2 - 64x - 72$

$0 = x^2 - 8x - 9 \quad 0 = (x-9)(x+1) \quad x = 9$

13. Continued



If $x=9$, then $y = \frac{50}{9-4} + 8 = 18$

The area of the paper is a minimum of

$$A(9) = \frac{50 \cdot 9}{9-4} + 8 \cdot 9 = \frac{450}{5} + 72 = 90 + 72 = 162 \text{ in}^2$$

with $x = 9$ in and $y = 18$ in since A' changes from Neg. to Pos. at $x = 9$ in.

The printable region has dimensions 5 in. by 10 in.